The purpose of this section is to briefly introduce Dijkstra’s algorithm to a reader that has some experience on computer programming, some experience on theoretical optimization, but none on numerical optimization. The general problem to be solved is to find the fastest way of connecting a source vertex with a destination vertex using the best possible combination of the edges of a grid. This problem is known as the “Shortest Path Problem”. The edges of the grid each one of them have source and destination vertices, and travel times associated to them. To illustrate the problem, see figure 5 below:

Figure 5: Vertices, edges, and travel times

Defining $V$ the set of vertices (A, B, C, D, E, and F), $E$ the set of edges (AB, BA, BC, CB, BD, DB, CD, DC, CE, EC, DE, ED, EF, and FE), $c_{ij}$ the travel time between vertex $i$ and $j$ using a direct edge (4, 4, 2.5, 2.5, 5, 5, 1.2, 1.2, 5, 5, 2.5, 2.5, 2, and 2 respectively), $x_{ij}$ as the number of times that edge $ij$ is used as part of an optimal path, and $N$ is the number of vertices (6), we have that the mathematical problem to be solved to find the shortest paths that start in vertex A to all the other
\( N - 1 \) vertices\(^{21}\) is the following:

\[
\begin{align*}
\min_{x_{ij}} & \quad \sum_{(i,j) \in E} c_{ij}x_{ij} \\
\text{s.t.} & \quad \sum_{j : (A,j) \in E} x_{Aj} - \sum_{j : (j,A) \in E} x_{jA} = N - 1 \\
& \quad \sum_{j : (i,j) \in E} x_{ij} - \sum_{j : (j,i) \in E} x_{ji} = -1, \forall i \in V \setminus \{A\}
\end{align*}
\]

This is a computationally intensive problem, and for grids with a large number of vertices and edges it might take some time, even for a modern computer. A simplification of the problem is Dijkstra’s algorithm, whose solution is the same as the one of equation 21, but takes much less computational resources. Dijkstra’s algorithm was invented in 1956 by Edsger W. Dijkstra, and the pseudocode (algorithm) is in table 3, where the only new piece of notation is \( n(i) \), the set of vertices that are one edge away from vertex \( i \). This algorithm has a complexity of \( O(N^2) \). This means that the number of computational operations grows as a polynomial of degree 2 in number of vertices.

\(^{21}\)If only one destination vertex is needed, the mathematical problem is similar, but has the same complexity as the one for \( N - 1 \) vertices, so in general it is preferred to write the problem for the whole set of destination vertices.
Table 3: Dijkstra’s algorithm pseudocode

\[
S \leftarrow \emptyset \\
\hat{S} \leftarrow V \\
d(i) \leftarrow \infty \ \forall i \in V \\
d(A) \leftarrow 0; \ \text{pred}(A) \leftarrow 0 \\
\text{while } |S| < N \text{ do} \\
\quad \text{let } i \in \hat{S} \text{ be a node such that } d(i) = \min_{j \in \hat{S}} d(j) \\
\quad S \leftarrow S \cup i \\
\quad \hat{S} \leftarrow \hat{S} - i \\
\quad \text{for all } e \in n(i) \text{ do} \\
\quad \quad \text{if } d(i) + c_{ij} < d(j) \text{ then} \\
\quad \quad \quad d(j) \leftarrow d(i) + c_{ij} \\
\quad \quad \quad \text{pred}(j) \leftarrow i \\
\quad \text{end if} \\
\quad \text{end for} \\
\quad \text{mark current as visited} \\
\quad \text{current} \leftarrow \arg\min_{v \in \text{unvisited}} \text{dist}[v] \\
\text{end while}
\]

Now we will illustrate the two steps required in our analysis of section 2 to calculate optimal travel times between every two locations in Mexico. The first thing to do was to obtain the distance between every pair of vertices that share an edge. For the purposes of this example, there are 6 vertices and 14 edges (see figure 6). The distance between vertices is written next to each edge. Then, using the digitized data set of Mexican roads, we obtain the speed of the edge that connects each pair of vertices. The speeds are colored in figure 6 too.
Combining the data of distances and speed, it is easy to obtain the travel times for each edge (see figure 7). This matrix of travel times is the input for Dijkstra’s algorithm, and it is easily verifiable that the optimal travel time from source A is the first row of table 4 (that is, the solution of equation 21). This table has the entire optimal travel time matrix, that is, the solution obtained from running Dijkstra’s algorithm 6 times.
Table 4: Optimal travel time matrix

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>4</td>
<td>6.5</td>
<td>7.7</td>
<td>10.2</td>
<td>12.2</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>0</td>
<td>2.5</td>
<td>3.7</td>
<td>6.2</td>
<td>8.2</td>
</tr>
<tr>
<td>C</td>
<td>6.5</td>
<td>2.5</td>
<td>0</td>
<td>1.2</td>
<td>3.7</td>
<td>5.7</td>
</tr>
<tr>
<td>D</td>
<td>7.7</td>
<td>3.7</td>
<td>1.2</td>
<td>0</td>
<td>2.5</td>
<td>4.5</td>
</tr>
<tr>
<td>E</td>
<td>10.2</td>
<td>6.2</td>
<td>3.7</td>
<td>2.5</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>12.2</td>
<td>8.2</td>
<td>5.7</td>
<td>4.5</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Market Access: a Four Region Example

Assume that this economy consists of four regions: A, B, C, and D. These four regions are connected by four roads with the direct travel times as illustrated in the left part of Figure 8. Notice there are multiple ways of going to and from any region of the economy. Running Dijkstra’s algorithm over this transportation network yields the 4x4 optimal travel times matrix (in hours) on the right of the same figure.

Figure 8: Four regions and four roads

For now, let’s focus on region A. This region’s market access is directly proportional to the GDP of the 4 regions, and inversely proportional to the travel time. For same values of GDP, region A’s market access is affected first by its own GDP (no
discount), then by B’s (a discount of 37%), C’s (a discount of 38%), and then D’s (a discount of 39%). Another way to look at the extent of this discount, is that for the same transport costs, in order for every region to be exactly 1/4 of region A’s market access, it must be that the GDP of regions B, C, and D to be larger than region A’s by 59%, 61%, and 64% respectively.

Now let’s add a fifth road to this economy: a fast, direct link from region A to region C, as pictured on the left of figure 9. The new optimal travel times are on the matrix on the right. Note that for region A, the optimal travel time to region C has decreased, but also, because of the triangle inequality, so has the optimal travel time to region D. Assuming that the GDP of none of the regions changes (short run), then, from region A’s perspective, the discounting from region C goes from 38% to 37%, and from region D goes from 39% to 38%. The market access of region A went up. And that’s it. That is all we measure in this paper. Just as described in this example, this paper will only measure the increment in the market access of region A due to the reduction in the row corresponding to A of the optimal travel times matrix.

Market access will not give any information on possible long-run outcomes such as the following:

1. Region B is not part of the trade route between A and C anymore. This means region B lost some market power. In the long run equilibrium, one might expect some migration from B to A, C, and even D. In the short-to-medium run, a reduction in the wages of region B is expected. This was partially calculated in section 5, because we assumed the factors of production fixed, so it was a short run analysis.

\[ \text{22Using the values of } F, \lambda, \text{ and } \theta \text{ from Section 3.} \]
2. The road that directly connects A and D is abandoned. The road that connects B and C has a drastic reduction in flow because A no longer trades with C using that road. The road that connects C and D has an increase in flow because now A trades with D using that road. This paper will not study congestion, including lane capacity, or possible changes in travel times due to congestion. It will also not study the volume of the flows and compare it to theoretical capacity of every type of road.

Figure 9: Four regions and five roads

Complementary Figures and Tables

In this section, we depict the 21 categories used for the road classification, as well as the calibrated speeds.
Table 5: Speed by Category of Infrastructure

<table>
<thead>
<tr>
<th>Category</th>
<th>Lanes</th>
<th>Speed (km/hr)</th>
<th>Category</th>
<th>Lanes</th>
<th>Speed (km/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal Toll</td>
<td>5+</td>
<td>90</td>
<td>State Free</td>
<td>6</td>
<td>90</td>
</tr>
<tr>
<td>Highways</td>
<td>4</td>
<td>85</td>
<td>Highways</td>
<td>4</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>70</td>
<td></td>
<td>3</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>50</td>
<td></td>
<td>1-2</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>40</td>
<td></td>
<td>5+</td>
<td>90</td>
</tr>
<tr>
<td>Federal Free</td>
<td>6+</td>
<td>90</td>
<td>State Toll</td>
<td>4</td>
<td>85</td>
</tr>
<tr>
<td>Highways</td>
<td>4</td>
<td>85</td>
<td>Highways</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>70</td>
<td>Urban Roads</td>
<td>N/A</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>1-2</td>
<td>50</td>
<td>Maritime Routes</td>
<td>N/A</td>
<td>35</td>
</tr>
<tr>
<td>Rural Pathways</td>
<td>N/A</td>
<td>3</td>
<td>Unpaved Roads</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Rest of the territory</td>
<td>N/A</td>
<td>2</td>
<td></td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Note: The only aspect that was calibrated in these categories was the speed as a function of infrastructure. Other variables such as highway capacity, or truck capacity as a function of the road were not specifically included in this analysis throughout the paper. However, this effect is captured in the estimation of the transportation cost function, as discussed in section 4.

Additionally, we map the travel times from Mexico City to every other location in Mexico, as well as every location in Mexico to the nearest border crossing to the U.S., and maritime ports, before and after the construction of the highways. In order to determine the major border crossings and ports depicted in figure 13 and figure 14, we obtain data from the U.S. Department of Transportation and the Secretaria de Comunicaciones y Transportes from Mexico respectively.

Major border crossings are ranked by the number of trucks passing through in 2010, and the 7 selected account for the 90.7% of the total. From west to east, the crossings are: Otay Mesa, Calexico, Nogales, Paso del Norte, Laredo, Hidalgo and Brownsville. The corresponding Mexican names are Tijuana, Mexicali, Nogales, Ciudad Juárez, Nuevo Laredo, Reynosa, and Matamoros. Major ports are defined by the volume of exports and imports (not including petroleum) in 2010, and the 8 selected account for 91.1% of the total. From west to east, the ports are: Ensenada, Guaymas, Manzanillo, Lázaro Cardenas, Altamira, Veracruz, Coatzacoalcos, and
Figure 10: Travel time from Mexico City to every other location in Mexico

Note: This diagram depicts the optimal travel times and not the routes, although they can be partially inferred from the travel times. The picture corresponds to one row of the matrix of size 1,977,537\times1,977,537 with the travel times. Notice how the three ferries in Baja California are an important source of connection to the rest of the country.

Punta Venado. Interestingly enough, neither Mazatlán nor Tuxpan are major ports under this measure.
Figure 11: Municipal population and municipal income per capita, 2010
Figure 12: Market access to national products for each of the 1,977,537 locations of the grid.
Figure 13: Travel time to the closest major border crossing to the United States

Notes: Top: baseline case. Bottom-left: baseline incorporating Durango-Mazatlán. Bottom-right: baseline incorporating Mexico City-Tuxpan. The top case has representations of the new highways to visualize any changes in the identity of the closest border due to the construction of the highways. The borders are, from west to east: Tijuana, Mexicali, Nogales, Ciudad Juárez, Nuevo Laredo, Reynosa, and Matamoros.
Figure 14: Travel time to the closest major sea port

Notes: Top: baseline case. Bottom-left: baseline incorporating Durango-Mazatl-n. Bottom-right: baseline incorporating Mexico City-Tuxpan. The top case has representations of the new highways to visualize any changes in the identity of the closest port due to the construction of the highways. The ports are, from west to east: Ensenada, Guaymas, Manzanillo, Lazaro Cardenas, Altamira, Veracruz, Coatzacoalcos, and Punta Venado. Notice how Mazatl-n’s closest port used to be Guaymas and with the new highway it is Manzanillo.