Implications of Brazilian institutional guidelines on educational efficiency

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ABSTRACT
This paper aims to investigate the relation between inefficiency in the Brazilian education system and municipal wealth, discussing how the actual legislation possibly influences it. To that end, we apply a stochastic frontier model which accommodates covariates in the asymmetric error component to analyze the impact of the GDP per capita on the inefficiencies. This methodology is applied to a dataset from the Rio Grande do Sul municipalities for the years of 2007 and 2017. The results indicate a positive effect, suggesting that richer municipalities are less efficient in allocating their resources.

KEYWORDS
Inefficiency; Fundeb; stochastic frontier analysis; Bayesian inference

JEL Classification: I20, H75, D61, C11.
1. Introduction

The basic education is, albeit partially, provided by different governments around the world. This investment is funded by taxpayers and, consequently, it is associated to the productive capacity to generate wealth, it means, the Gross Domestic Product (GDP) per capita. Considering its relevance and due to the limited resources available in the public administration that also supplies services like healthcare, law enforcement, security and others, it is important to ensure an efficient allocation of this capital.

In education economics literature, there is empirical evidence which supports that an increase in financial funding assigned to education does not necessarily imply a better performance in standardised assessments of educational attainment (Hanushek and Luque 2003; Glewwe et al. 2011; Monteiro 2015). However, the Education at a Glance 2017 (OECD 2017) presents an association between developed countries from the Organisation for Economic Co-operation and Development (OECD) and better results in the PISA (Programme for International Student Assessment) test.

As a result, it is possible to infer that school achievements is not only related to the total amount available, but also to an efficient allocation of it. Afonso and Aubyn (2006) explore these ideas and propose a two-stage approach in which the efficiencies are obtained in the first step and, in the second, they estimate the relationship between the previous results and the GDP per capita. From PISA data of twenty five OECD countries, the authors conclude that the efficiency is strongly related to GDP, in other words, the richer a country is, more efficient it is in providing a better education.

Nonetheless, is this relationship also observed for municipalities within a country? Oliveira and Santos (2005) evaluated the Portuguese schools efficiencies and analyzed the influence of the GDP per capita of municipalities where they were located, obtaining a not significant result. In the Brazilian case, this question is particularly interesting because the municipalities face severe fiscal restrictions and major challenges in the area. In addition, article 212 of the Constitution of the Federative Republic of Brazil establishes that municipalities must allocate at least 25% of their budget revenue on public education.

Therefore, the main hypothesis of this paper is: wealthy municipalities are less efficient in the allocation of their resources due to the obligation of investing an amount that is possibly higher than the necessary. In order to confirm our hypothesis, we intend to investigate the relation between the GDP per capita of municipalities and their inefficiencies. To that end, an extension that accommodates covariates in the asymmetric error component of the spatial stochastic frontier model introduced by Schmidt et al. (2009) is proposed. The methodology is applied to a dataset from the Rio Grande do Sul municipalities for the years of 2007 and 2017.

The state is located in the southern region of Brazil and, compared to others, this region has historically a high level of education as consequence of its European heritage. Considering its economic capacity, territorial dimension and population size, it is one of the most representative Brazilian states. In spite of its prominent position, it has been losing performance in educational achievements. In 2007, it was ranked as the 5th public education system in Brazil with an IDEB of 3.7, falling 6 positions ten years after with an IDEB of 4.4. The state presented an improvement of 19% in the period, while the national average was 26% (INEP 2017). Despite its privileged position as the 5th highest GDP per capita in 2017, over the last years, Rio Grande do Sul has been facing a fiscal crisis caused by the debt with the Union and the repeated deficits.

The remainder of the paper is organized as follows. Section 2 brings a brief literature
review of the main methods to measure efficiency in education economics, underlining some interesting outcomes. Additionally, it outlines the Brazilian legislation and its connection with the concept of adequacy in school finance. Section 3 introduces the methodology and details the inference process. In Section 4, the dataset and the results are presented and discussed. At last, Section 5 brings the main conclusions.

2. Background

2.1. Literature Review

“What matters more are the choices countries make in how to allocate that spending and the policies they design to improve the efficiency and relevance of the education they provide” (OECD 2013). In this sense, Angel Gurria, OECD Secretary-General, underlines the importance of an efficient public spending and a rational allocation of these resources. Since education is relevant for promoting several outcomes, such as cognitive and non-cognitive skills and economic growth (Cunha, Heckman, and Schennach 2010; Hanushek and Kimko 2000), this is a topic of intense debate among policy makers, teachers and other stakeholders.

The analysis of education provision efficiency consists in defining a technology function to produce knowledge, which represents the maximum output that can be achieved given a provision. Then, a system is considered efficient if its producers make an effective use of available inputs. In an inefficient system, there is a possibility of increasing attainments for a given spending level, or decreasing expenditure for given attainments (De Witte and López-Torres 2017).

However, defining and estimating a production function is not a trivial task, once it is necessary to specify the relevant inputs. Glewwe et al. (2011) reviewed the literature about school resources and educational outcomes in developing countries and concluded that most schools and teachers characteristics are not statistically significant to explain the learning process. In addition, the results are influenced by several factors that are beyond the control of the evaluated observation. Coleman et al. (1966) observed that investments explain only 10% of academic achievements, while the remainder percentage depends on other economic variables and students family environment, which are known as non-discretionary variables.

Therefore, different specifications and methods have been applied to study the importance of structural, institutional and socioeconomic variables on educational achievements and efficiency scores. Nonetheless, it is possible to identify two main modeling techniques that are implemented in the literature, the first is known as the Data Envelopment Analysis (Charnes, Cooper, and Rhodes 1978, DEA) and the second as Stochastic Frontier Analysis (Aigner, Lovell, and Schmidt 1977; Meeusen and van Den Broeck 1977, SFA). Commonly, both techniques are employed as a first step in an empirical strategy based on a two-stage procedure in which the second stage lies on a regression-type model between the efficiency scores and explanatory variables.

Bradley, Johnes, and Millington (2001) and Worthington (2001) provide a list of studies conducted in several countries that illustrates different applications of DEA methodology. Agasisti (2013), for example, measures the performance of Italian secondary schools, investigates which factors affect efficiency through a Tobit regression and concludes that there is a potential role for better results by increasing competition. Considering the SFA methodology, a broad literature was also developed (Izadi et al. 2002; Lenton 2008; Kuo and Ho 2008). For instance, Lewis, Pattinasarany, and Sahn
(2011) analyzed the public elementary schools in Indonesia and the results suggest that the outcomes might be enhanced even with a reduction in total spending.

Furthermore, there are some alternative methods. Deutsch, Dumas, and Silber (2013) applied the corrected least squares method (Richmond 1974) to estimate the efficiency of five Latin American countries and obtained that individual efficiency is likely to be influenced by increments in public debt caused by expansions in education access. Thieme, Giménez, and Prior (2012), on its turn, used directional distance functions (DDF) to evaluate Chilean urban schools and identified that the most important source of inefficiency is the resource endowment effect. The authors also argued that when specific variables concerning the amount allocated are disregarded, the performance is undervalued.

Regarding the Brazilian case, Carvalho and Sousa (2014) and Gonçalves and França (2013) applied DEA methodology to a dataset from Brazilian municipalities, and northeastern and southeastern public schools respectively. The first paper indicated that, even discounting the environmental factors, improvements can be made. The second established a positive relation between efficiency gains and decentralized management. Adopting an approach based on quantile estimators, Oliveira, Souza, and Annegues (2018) suggest that management autonomy is not a determining factor for efficiency degrees in Brazilian public schools. Ferraz, Finan, and Moreira (2012), on the other hand, looked to the allocation of resources problem and student outcomes through the corruption perspective. The authors examined if missing resources due to corruption affect the performance based on the variation of its incidence across municipalities. The findings suggest a significant negative impact on primary school students achievements.

It is important to mention that literature is not convergent, and conclusions vary according to method, period and country analyzed. Kirjavainen (2012) fitted different stochastic frontier models for panel data to estimate a production function and the efficiency of Finnish general upper secondary schools. The estimates pointed that inefficiency and rankings based on their scores diverge considerably depending on the type of the applied model.

2.2. Adequacy in school finance and Brazilian legislation

Adequacy in school finance is a term implemented in education economics to define the amount of funding required to produce a desired level of student performance. According to Odden (2003), determining sufficient revenue levels involves the following steps: identifying the costs of effective programs and strategies, converting these investments into appropriate school finance structures, certifying that the resources are used in schools to produce the aspired results. Ruggiero (2007) highlights that these levels vary in accordance with the socioeconomic characteristics of municipalities. For example, locations in which pupils encounter precarious conditions should invest more.

The concept introduced above is applied on the design of public policies in order to guarantee a minimum expenditure in education (Hanushek 1994). The post-1990 school finance reforms in the United States, for example, were strongly grounded on adequacy concepts, and a topic of concern is its impact on absolute and relative spending and achievement in low-income school districts. Lafortune, Rothstein, and Schanzenbach (2018) demonstrate that it led immediate and sustained increases in spending in these districts, and find that the reform had a large positive impact in the achievement of students. Lee (2012) focus on assessing the achievement gap in
mathematics proficiency standard from adequacy and equity perspectives, and found that the required school funding varies by poverty status.

In Brazil, the re-democratization process in the 1980s, promoted several reforms in legislation and financial system of public education. The article 212 of the 1988 Constitution of the Federative Republic of Brazil establishes that states and municipalities must assign at least 25% of their budget revenue on the maintenance and development of the basic public education system. The Law No. 9394 of 1996 defines the “Guidelines and Bases of National Education” (LDB) and regulates this system, for example, detailing through the articles 70 and 71 how municipalities must and must not invest their resources. From the same year, the Law No. 9424 created a specific fund to education that aimed to guarantee a minimum investment per pupil and promote a resources distribution across municipalities within the same state. This fund was initially focused on elementary and middle school (Fundef - Fund for Elementary and Middle School and for Enhancing the Value of the Teaching Profession).

However, in 2007, Fundef was replaced by Fundeb (Fund for Basic Education and for Enhancing the Value of the Teaching Profession) with the promulgation of the Law No. 11494, which extended it to all basic education system, including kindergarten, high school and basic education for adults who do not access it at regular age. The Fundeb consists of a state account in which the municipalities deposit 20% of the revenue collected from eight specific taxes. Consequently, to accomplish the 25% defined by the federal constitution, at least others 5% of the amount collected from the same eight taxes must be allocated in an account specific to education. Moreover, at least 25% of the revenue collected from the remaining taxes must be saved at the same account. Finally, the articles 70 and 71 of the LDB limit how resources from this account are spent and the articles 21, 22 and 23 of the Law No. 11494 define how Fundeb is invested.

The Fundeb’s state account is redistributed across municipalities according to the number of pupils enrolled in public schools, ignoring other municipalities sources for school funding. Consequently, both wealthy and poor municipalities receive a similar amount per pupil. Based on a socioeconomic indicator ranking, Bertoni et al. (2018) showed that Fundeb represents almost the same per pupil spending relative share in all municipalities. According to the authors, it means a neutral rule of transference that is not sufficient to equalize municipal resources, once the more developed municipalities raise more own funds than the lower income ones.

Towards this scenario and the extensive literature about inefficient allocation of resources in education summarized in Section 2.1, Monteiro (2015) evaluated the impact of higher spendings observed on the oil producing municipalities that were benefited by higher revenues from royalties over the period of 2000 through 2010. The author concluded that an increase of 15% on revenues and, in consequence, on education funding was not converted into better results in comparison with other municipalities from the Brazilian coast. Therefore, it is plausible to think that locations where larger GDP per person are observed have less incentive for an efficient management given the current legislation.

3. Methodology

Suppose that observations are available in the form of balanced panel data for $N$ municipalities across $T$ times. Let $y_{it}$ be the logarithm of the output of the municipality
on time $t$, the stochastic frontier model is defined by the following equation

$$y_{it} = g(r_{it}, \theta) - u_{it} + \epsilon_{it},$$

(1)

where $g(r_{it}, \theta)$ is the production function, $r_{it}$ is a vector of inputs, and $\theta$ is a vector of parameters that describe the effect of each input on the output $y_{it}$. The component $u_{it}$ follows an asymmetric positive distribution and models the inefficiency of unit $i$ on time $t$. The random error, $\epsilon_{it}$, is assumed independent of $u_{it}$ and follows a Gaussian distribution centred at zero with variance $\sigma^2$, that is, $\epsilon_{it} \sim N(0, \sigma^2)$.

Considering the distribution of the inefficiency component, there are different proposals in the literature: the exponential (Meeusen and van Den Broeck 1977), the half-normal (Aigner, Lovell, and Schmidt 1977), the truncated normal (Stevenson 1980), the gamma (Greene 1990). Here, the truncated normal distribution is adopted and its mean is a function of municipality effects and covariates. More specifically, we have

$$u_{it}|\alpha_i, z_{it}, \eta, \tau^2 \sim N^+(\mu_{it}, \tau^2)$$

(2)

$$\mu_{it} = \alpha_i + z_{it}\eta$$

(3)

where $N^+(a, b)$ denotes the normal distribution truncated at zero, whose associated normal has mean $a$ and variance $b$.

The above specification is similar than the one introduced by Schmidt et al. (2009), the difference consists in the possibility of modeling the inefficiency not only as a function of $\alpha = (\alpha_1, \ldots, \alpha_N)$ but also of covariates. In accordance with Schmidt et al. (2009), $\alpha_i$ is allowed to represent a process that spreads through spatial contagion, such as social and economic conditions. This process is frequently represented by priors that vary smoothly across space and, in several applications, it is assumed that $\alpha$ follows a conditional autoregressive distribution which depends on its neighbors. Therefore, this specification enables that the spatial structure is naturally imposed in the model (Besag, York, and Mollié 1991).

Our prior belief about this structure is motivated by empirical evidence presented in Power and Rodrigues-Silveira (2019). The authors calculate a measure of vote-revealed ideology called Municipal Ideological Score (MIS) over the course of 13 electoral cycles between 1994 and 2018, and results suggest that nearby municipalities share similar ideologies. Therefore, we are making a prior assumption that these similarities have an impact on the educational policy and governance mechanisms adopted by the elected politicians. Moreover, we can alternatively interpret the latent effects $\alpha_i$ as an attempt to capture these non observable particularities of each municipality.

### 3.1. Inference procedure

Let $y = (y_{11}, \ldots, y_{1T}, \ldots, y_{N1}, \ldots, y_{NT})'$ be a random sample of the logarithm of the outputs and $u = (u_{11}, \ldots, u_{1T}, \ldots, u_{N1}, \ldots, u_{NT})'$ be the vector of unobserved inefficiencies. Assuming the model presented by Equations (1)-(3), the likelihood function
is given by

\[
f(y, u | r, \theta, \sigma^2, \alpha, \eta, \tau^2) \propto (\sigma^2)^{-N_T} \exp \left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it} - ru_i \theta + u_{it})^2 \right\} \\
\times (\tau^2)^{-N_T} \exp \left\{ -\frac{1}{2\tau^2} \sum_{i=1}^{N} \sum_{t=1}^{T} (u_{it} - \alpha_i - z_{it} \eta)^2 \right\} \mathbb{1}_{\{u_{it}>0\}}. \]

Performing a Bayesian analysis, an important step is the prior distribution selection, in particular, in this case, the prior distribution of the latent effects \( \alpha \). Since the data consists of observations made across municipalities, a certain spatial correlation is expected from these effects and due to this geographical component, it is intuitive to think that the inefficiencies from neighboring municipalities share some common characteristics. For these reasons, as in Schmidt et al. (2009) and following (Besag, York, and Mollié 1991), we assume a conditional autoregressive (CAR) prior for \( \alpha \).

The conditional autoregressive (CAR) prior distribution is described as

\[
p(\alpha | \psi^2) \propto \exp \left\{ -\frac{1}{2\psi^2} \sum_{i=1}^{N} \sum_{j<i} W_{ij} (\alpha_i - \alpha_j)^2 \right\}, \tag{4}
\]

and it is denoted by \( \alpha \sim CAR(\psi^2) \). The matrix \( W \) is an adjacency matrix, and since the spatial phenomenon observed in Power and Rodrigues-Silveira (2019) is not coincident with any regional division for the state of Rio Grande do Sul, we assume a standard specification in which \( W_{ij} = 1 \) if municipality \( i \) shares a border with municipality \( j \) and \( W_{ij} = 0 \) otherwise. Additionally, we also assume two other specifications in which \( W_{ij} = 1 \) if municipality \( i \) belongs to the same immediate/intermediate region than municipality \( j \) and \( W_{ij} = 0 \) otherwise. Both immediate and intermediate regional divisions are defined by IBGE following a criteria based on urban networks. The distribution in Equation (4) is an improper joint distribution for \( \alpha \) in a sense that it is possible to add a constant to all \( \alpha_i \) without affect it (Banerjee, Carlin, and Gelfand 2004). In order to guarantee that the posterior is proper, each sample from \( \alpha \) obtained through Markov Chain Monte Carlo (Gamerman and Lopes 2006, MCMC) methods is centered (Besag and Kooperberg 1995; Gelfand and Sahu 1999).

Thus far we have discussed the prior distribution of the latent effects \( \alpha \). However, from a Bayesian perspective, the model specification is complete only after assigning a prior distribution to all unknowns in the model. Thus, it is remaining to talk about the prior distribution of the others parameters. Let \( \vartheta \) be the parametric vector, \( \vartheta = (\theta, \sigma^2, \alpha, \eta, \tau^2, \psi^2) \), and assume that all of its components are independent a priori. Hence, the joint prior distribution for \( \vartheta \) is given by

\[
p(\vartheta) = \prod_{j=1}^{p} [p(\theta_j)p(\sigma^2)p(\alpha | \psi^2)p(\psi^2)] \prod_{k=1}^{q} [p(\eta_k)] p(\tau^2). \tag{5}
\]

In this paper, we follow a conjugate prior analysis. Therefore, considering the coefficients \( \theta_j, j = 1, \ldots, p \) and \( \eta_k, k = 1, \ldots, q \), a normal prior distribution, \( N(\mu_0, \sigma_0^2) \), in which the hyperparameters \( \mu_0 = 0 \) and \( \sigma_0^2 = 100 \) were specified. For the scale parameters \( \sigma^2 \) and \( \tau^2 \), an inverse gamma prior distribution, \( IG \sim (\phi, \phi) \), with \( \phi = 0.01 \) was chosen. A special care must be taken when assigning the prior distribution for
$\psi^2$, as this is a non-identifiable parameter in the sense of Dawid (1979), it is not recommended to be too uninformative (Besag and Kooperberg 1995). For this reason, we adopt the same strategy as Schmidt et al. (2009), and an inverse gamma prior distribution, $IG(\phi_0, \phi_0)$, in which the mean is equal to the OLS variance estimate based on an independent stochastic frontier model and the variance is fixed. From the conjugate prior analysis, we obtain full conditional posterior distributions in closed form. Therefore, we use a MCMC algorithm based on the Gibbs sampler (Gelfand and Smith 1990) and its description step by step is presented in Appendix A.

4. Empirical Analysis

4.1. The Data

Three different databases were used, the school census from the National Institute for Educational Studies and Research “Anísio Teixeira” (INEP), SIDRA from the Brazilian Institute of Geography and Statistics (IBGE) and the National Treasury Secretariat database. From the first, we obtained the municipalities Basic Education Development Index (IDEB) for the students in 9th grade of lower secondary school, the infrastructure available in schools index and the pupil-teacher ratio. From the second, we collected the GDP of municipalities. From the last, we accessed the amount of Fundeb resources designated for each municipality.

As mentioned in Section 1, our study focus on Rio Grande do Sul and, therefore, our data consists on its municipalities. The analysis contemplates the years of 2007 and 2017, both are representative since in the first the Fundeb was established by the Law No. 11494, and the second represents a decade of its implementation. Furthermore, the number of missing information for these two years is not significantly high, and provides a final dataset composed by 445 municipalities in a total of 497. An alternative analysis with a more extended panel, for example, a biannual panel from 2007 to 2017, was also possible, however due to a large number of missing data, our sample would be reduced to a small number of municipalities.

In our analysis, the IDEB is specified as output. This index is a product of two variables that evaluate the education quality: the proficiency in Mathematics and Portuguese language, and the passing rate. In consequence, a municipality is considered efficient not only by its grade but also by its capability of graduating students from lower secondary school. This choice is made because an educational system in which students systematically fail is not desirable. On the other hand, high approval rates are possibly correlated with insufficient learning from a sort of pupils.

As inputs of our model, we have the following variables: the teacher-pupil ratio, the infrastructure index and the Fundeb. The teacher-pupil ratio represents the labour input in our production function. The infrastructure index consists of the total resources available in the schools, it means, sports facilities, science and computer laboratories, libraries, internet access, projector and others. This variable, on its turn, serves as the physical capital. The Fundeb resources designated for each municipality was normalized by the total number of students registered according to the school census and it is considered as a variable for the public education spending. Table 1 summarizes the descriptive statistics.
Table 1. Summary statistics.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>OBS.</th>
<th>MEAN</th>
<th>STD</th>
<th>MIN.</th>
<th>MAX.</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDEB</td>
<td>935</td>
<td>4.35</td>
<td>0.70</td>
<td>2.50</td>
<td>6.70</td>
</tr>
<tr>
<td>Teacher-pupil ratio</td>
<td>992</td>
<td>0.29</td>
<td>0.07</td>
<td>0.13</td>
<td>0.68</td>
</tr>
<tr>
<td>Fundeb</td>
<td>991</td>
<td>1819.13</td>
<td>911.01</td>
<td>71.82</td>
<td>4700.70</td>
</tr>
<tr>
<td>Infrastructure</td>
<td>992</td>
<td>6.88</td>
<td>0.96</td>
<td>3.34</td>
<td>10.00</td>
</tr>
<tr>
<td>GDP</td>
<td>993</td>
<td>29083.06</td>
<td>21398.81</td>
<td>7711.18</td>
<td>393569.40</td>
</tr>
</tbody>
</table>

Afonso and Aubyn (2006) state: “We have considered the option of using education spending per student as an input. However, results would be hardly interpretable, as they would reflect both inefficiency and cost provision differences. For example, countries where teachers are better paid would tend to show up as inefficient, irrespective of the intrinsic performance of the education system”. Therefore, the choice for the Fundeb as an input and not the total education spending seems a good option since both variables are highly correlated and the first is not affected by differences in teachers remuneration for example.

4.2. Results

Assuming the model proposed in Section 3, we consider three different specifications to the adjacency matrix $W$: the first is based on municipalities which share a border (M1); the second, on municipalities that belong to the same immediate region (M2); the third, on municipalities that belong to the same intermediate region (M3). We adopt the Watanabe-Akaike information criterion (Watanabe 2011, WAIC) as the criteria for model selection. It is defined as function of the posterior predictive density and a correction for the effective number of parameters to adjust for overfitting, and smaller values of WAIC indicate better fit. Details are provided in the Appendix B.

Table 2. Mean and Highest Posterior Density (HPD) interval of the parameters based on a sample of size of 1000 from the posterior distribution for all three models and their respective WAIC.

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$ (Intercept)</td>
<td>1.6881 (1.4940,1.9091)</td>
<td>1.5903 (1.3896,1.7719)</td>
<td>1.7791 (1.5897,1.9870)</td>
</tr>
<tr>
<td>$\theta_1$ (Dummy2017)</td>
<td>0.1455 (0.1214,0.1695)</td>
<td>0.1416 (0.0755,0.1579)</td>
<td>0.1297 (0.1297,0.1796)</td>
</tr>
<tr>
<td>$\theta_2$ (Teacher-pupil ratio)</td>
<td>0.1087 (0.0613,0.1429)</td>
<td>0.1160 (0.0755,0.1579)</td>
<td>0.1448 (0.1003,0.1852)</td>
</tr>
<tr>
<td>$\theta_3$ (Fundeb)</td>
<td>-0.0043 (0.0272,0.0196)</td>
<td>-0.0041 (-0.0262,0.0182)</td>
<td>-0.0253 (-0.0482,-0.0011)</td>
</tr>
<tr>
<td>$\theta_4$ (Infrastructure index)</td>
<td>0.0790 (0.0229,0.1371)</td>
<td>0.1320 (0.0682,0.1813)</td>
<td>0.1229 (0.0723,0.1925)</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.0061 (0.0034,0.0079)</td>
<td>0.0087 (0.0058,0.0111)</td>
<td>0.0067 (0.0039,0.0095)</td>
</tr>
<tr>
<td>$\eta_1$ (GDP)</td>
<td>0.0298 (0.0253,0.0388)</td>
<td>0.0247 (0.0200,0.0315)</td>
<td>0.0292 (0.0218,0.0360)</td>
</tr>
<tr>
<td>$\tau^2$</td>
<td>0.0029 (0.0014,0.0055)</td>
<td>0.0026 (0.0011,0.0053)</td>
<td>0.0029 (0.0013,0.0058)</td>
</tr>
<tr>
<td>WAIC</td>
<td>-1837.1550</td>
<td>-1513.624</td>
<td>-1692.243</td>
</tr>
</tbody>
</table>

Table 2 summarizes the results obtained from the inference process for M1, M2, and M3, presenting the posterior means and the 95% high posterior density credibility interval for the parameters. These results are obtained after running 100000 iterations from the MCMC scheme in Appendix A, discarding the first 20000 as a burn-in period, and storing only every 80th values in order to reduce the autocorrelation between
successive values of the simulated chain. Consequently, we have a final sample of 1000
draws. The WAIC suggests that the specification M1 presented the best fit, and from
now on the results and the analysis are focused on it.

Figure 1 shows the spatial effects $\alpha_i$. Figure C1 in Appendix C illustrates the inefficiencies $u_{it}$ for the years of 2007 and 2017 considering the municipalities located in state of Rio Grande do Sul under analysis. Interpreting figure C1, there is almost no variation on the inefficiencies, in other words, regions with higher levels of inefficiency have not improved significantly their investment policy along the years and a feasible explanation for that is the lack of incentives for better practices due to the actual legislation.

From Table 2, it is possible to observe that from the three inputs, only the Fundeb is
not significant since its credibility interval contains the 0. There is empirical evidence
in the literature supporting that an increase in financial funding assigned to education
does not necessarily imply a better performance in standardised assessments of edu-
cational attainment (Hanushek and Luque 2003; Glewwe et al. 2011). Respecting the
Brazilian case, Monteiro (2015) evaluated the impact of higher spendings observed on
the oil producing municipalities over the period of 2000 through 2010, concluding that
an increase on education funding was not converted into better results in comparison
with other municipalities from the Brazilian coast.

The infrastructure available in schools index has a positive and significant coefficient.
Looking for variables that contemplate school resources, there is also not a definite understanding about them and their effects on student performance (Glewwe et al. 2011). Card and Krueger (1996) observed that while most of the literature on test scores points to little, if any, effect of school resources, some observational studies and experiments have found a connection. Figlio (1999) argues that these differences may be attributable in part to the functional form assumptions of the school production function used in the existing literature.

In this work, we used the teacher-pupil ratio as long as it is broadly applied as input in similar contexts than ours (Afonso and Aubyn 2006; Kirjavainen 2012; Agasisti 2013). The estimates point that this ratio has also a positive and significant coefficient, it means that the supply of teachers contributes to a better educational system. De Witte and López-Torres (2017) interprets this result, explaining that larger supplies enable more individualized work with students.

The main hypothesis of this paper says that wealthy municipalities are less efficient in the allocation of their resources due to the legislation and mechanisms introduced in Section 2.2. To that end, we extended the model proposed by Schmidt et al. (2009) to accommodate the GDP as a covariate that explains the inefficiency. From Table 2, we observe that this variable has a positive relation with the inefficiency and it is statistically significant. This evidence endorses our hypothesis and it is in accordance with Monteiro (2015) who concluded that Brazilian oil producing municipalities benefited by revenues from royalties are less efficient than others with similar characteristics. Despite the fact that this result must be carefully interpreted assuming different states from the Rio Grande do Sul, it is an indication for a wider phenomenon. These points also demonstrate that the outcomes obtained by Afonso and Aubyn (2006), Fonchauny and Sama (2016) and Cordero, Santín, and Simancas (2017) about the relation between GDP and inefficiency in a country level might not be observed when we focus on municipalities, in particular, when we have a rigid legislation about the amount that must be invested in education.

Although interpretations at the national level should be done with caution, the joint analysis of our results possibly contributes for the debate about the current Fundeb’s design and its effectiveness. Since Fundeb’s validity expires at December 2020, policymakers and civil society organizations are discussing the possibility of making it permanent jointly with modifications on the strategy of resources distribution. As elucidated in Section 2.2, the actual distribution policy is mainly centered on the number of students enrolled in the basic public education system, not accounting, for example, for the special necessities that poor municipalities have. In the same section, we presented results that point the importance of allocating more resources in low income districts, and its positive impact on educational achievements and completed years of education (Lafortune, Rothstein, and Schanzenbach 2018; Jackson, Johnson, and Persico 2016). Then, our results point to a mechanism for Fundeb’s distribution based on a proportional allocation in which more resources should be directed for low income municipalities. This alternative would contribute for improving school results and efficiency in resource allocation.

5. Conclusion

A common idea in Brazilian public debate is that advances in educational quality are directly proportional to the amount of investment in the area. Although this argumentation might be appealing, the education economics literature presents some
evidences in a different direction (Hanushek and Luque 2003; Glewwe et al. 2011; Monteiro 2015), exposing the necessity to a well designed public policy and rigorous evaluations about its effectiveness. Towards the current economic scenario and the serious fiscal crisis that Brazil is facing, a particular topic of interesting rises from the discussions: efficiency in education management, specially, in education spending.

Therefore, this paper contributes to the literature investigating the relation between inefficiency in the Brazilian education system and municipal wealth, discussing how the actual legislation possibly influences it. We underline the actual legislation because it imposes rigid regulations that disregards the economic capacity of each municipality and does not introduces incentives for efficient policies which is of great importance since local governments have limited budgets. A stochastic frontier model was applied to a panel dataset from the municipalities of Rio Grande do Sul state over the years of 2007 and 2017, and the results indicated that the GDP per capita has a positive effect on inefficiencies, suggesting that richer municipalities are less efficient in allocating their resources and corroborating to our main hypothesis. In addition, no significant improvements on efficiencies were observed over the period under analysis, indicating a lack of incentives. For future research, this model might be applied to a larger number of municipalities, covering other regions of the country and making generalizations of our results feasible.

Appendix A. MCMC Algorithm

The posterior simulation method is based on a MCMC sampler following the steps below:

Step 1: Sample from the conditional distribution \( \theta | y, u, r, \sigma^2 \sim N_p(\mu_1, \Sigma_1) \), where

\[
\Sigma_1 = \left( \Sigma_0^{-1} + \frac{1}{\sigma^2} (r^T r) \right)^{-1} \quad \text{and} \quad \mu_1 = \Sigma_1 \left( \Sigma_0^{-1} \mu_0 + \frac{1}{\sigma^2} (r^T (y + u)) \right),
\]

in which \( \Sigma_0 = \sigma^2_0 I_p \) and \( \mu_0 \) is a \( p \)-dimensional vector of \( \mu_0 \).

Step 2: Sample from the conditional distribution \( \sigma^2 | y, u, r, \theta \sim IG(\phi_1, \phi_2) \), where

\[
\phi_1 = \phi + \frac{NT}{2} \quad \text{and} \quad \phi_2 = \phi + \frac{1}{2} (y - r \theta + u)^T (y - r \theta + u).
\]

Step 3: For \( i = 1, \ldots, N \) and \( t = 1, \ldots, T \), sample from \( U_{it} | y, u, \theta, \sigma^2, \alpha, z, \tau^2 \sim TN_{[0, +\infty)}(a_1, a_2) \), where

\[
a_1 = \frac{\sigma^2 (z_{it} \eta) + \tau^2 (y_{it} - r_{it} \theta)}{\sigma^2 + \tau^2} \quad \text{and} \quad a_2 = \frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2},
\]

in which \( TN_{[0, +\infty)}(\cdot, \cdot) \) is the truncated normal distribution over the interval \([0, +\infty)\).

Step 4: Sample from the conditional distribution \( \eta | u, \alpha, z, \tau^2 \sim N_q(\mu^* , \Sigma^*) \), where

\[
\Sigma^* = \left( \Sigma_0^{-1} + \frac{1}{\tau^2} (Z^T Z) \right)^{-1} \quad \text{and} \quad \mu^* = \Sigma^* \left( \Sigma_0^{-1} \mu_0 + \frac{1}{\tau^2} (Z^T (u - \alpha)) \right),
\]

in which \( \Sigma_0 = \sigma^2_0 I_q \) and \( \mu_0 \) is a \( q \)-dimensional vector of \( \mu_0 \).
Step 5: Sample from the conditional distribution $\tau^2 | u, z, \eta \sim IG(\phi_1^*, \phi_2^*)$, where

$$\phi_1^* = \phi + \frac{NT}{2} \text{ and } \phi_2^* = \phi + \frac{1}{2} (u - \alpha - \mathbf{z}\eta)^T (u - \alpha - \mathbf{z}\eta).$$

Step 6: For $i = 1, \ldots, N$, sample from $\alpha_i | u, z, \eta, \psi^2 \sim N(b_1, b_2)$, where

$$b_1 = \frac{\tau^2 \sum_{j \in J} \alpha_j + \psi^2 \sum_{t=1}^T (u_{it} - \mathbf{z}_i \eta)}{\tau^2 + \psi^2} \text{ and } b_2 = \frac{\tau^2 \psi^2}{\tau^2 + \psi^2},$$

in which $J$ is a set of index with the neighbors of $i$.

Step 7: Sample from the conditional distribution $\psi^2 | \alpha \sim IG(\hat{\phi}, \hat{\phi})$, where

$$\hat{\phi} = \phi_0 + \frac{NT - c}{2} \text{ and } \hat{\phi} = \phi_0 + \frac{1}{2} \sum_{i=1}^N \sum_{j<i} W_{ij} (\alpha_i - \alpha_j)^2,$$

in which $c$ is the number of blocks.

**Appendix B. Watanabe-Akaike information criterion**

Following Gelman, Hwang, and Vehtario (2014), the Watanabe-Akaike information criterion (WAIC) has an alternative adjustment as follows:

$$WAIC^* = 2 \sum_{i=1}^n \left[ \log \left( E_{(\theta|y)} p(y_i|\theta) \right) - E_{(\theta|y)} (\log(p(y_i|\theta))) \right].$$

Therefore, we have:

$$WAIC = -2 \left[ p(y) - WAIC^* \right],$$

in which $p(y) = \sum_{i=1}^n \log \int p(y_i|\theta) p(\theta|y) d\theta$.

In practice, $p(y)$ and $WAIC^*$ are calculated using the draws obtained from the posterior simulations, it means,

$$\bar{p(y)} = \sum_{i=1}^n \log \left( \frac{1}{T} \sum_{t=1}^T p(y_i|\theta^{(t)}) \right),$$

$$\bar{WAIC^*} = 2 \sum_{i=1}^n \left[ \log \left( \frac{1}{T} \sum_{t=1}^T p(y_i|\theta^{(t)}) \right) - \frac{1}{T} \sum_{t=1}^T \log \left( p(y_i|\theta^{(t)}) \right) \right].$$
Appendix C. Inefficiencies point estimates

Figure C1. Inefficiencies $u_{it}$ point estimates assuming M1: the upper figure illustrates $t = 2007$; the bottom figure, $t = 2017$. 
References


